UDC 531.36+62-50

ON ADAPTIVE STABILIZATION OF PROGRAMED MOTIONS OF MECHANICAL SYSTEMS

PMM Vol. 41, №5, 1977, pp. 859-869

A. R. GINZBURG and A. V. TIMOFEEV
(Leningrad)

(Received June 28, 1976)

The problem of stabilizing programed motions of mechanical systems with holonomic and nonholonomic relationships is considered for various degrees of information about parameters of the equation of motion. First, the laws of control that stabilize programed motions in the nonadaptive case when parameters of the equation of motion are known and the initial perturbations are arbitrary, are synthesized. The adaptive control is constructed on that basis; such control ensures after the transitional process of adaption the closeness of the actual and programed motions. Solution of the adaption problem is based on the method of finite-convergent algorithms for solving systems of inequalities proposed in [1]. Estimates of the time taken by the transitional process are presented for the adaptive and non-adaptive cases. Application of proposed algorithms for adaptive stabilization of programed motions of a transport robot on a caterpillar undercarriage and of a robot manipulator is discussed [2-5].

1. Statement of the problem. Let us consider a mechanical system defined by an equation of the form

$$A(q, q^{\cdot}, \xi)\, q^{\cdot\cdot} + b(q, q^{\cdot}, \xi) = u \tag{1.1}$$

where q is the n vector of the system generalized coordinates, and $q^{\cdot}$ and $q^{\cdot\cdot}$ are its first and second derivatives with respect to time; u is the n-vector of generalized forces which play the part of control; ξ is the r-vector of the system parameters; $A(q, q^{\cdot}, \xi)$ is an $n \times n$ nonsingular matrix-function, and $b(q, q^{\cdot}, \xi)$ is an n-vector function. The conditions of existence and uniqueness of solution of Eqs. (1.1) are satisfied, since system (1.1) consists of Lagrange equations of the second kind for which these conditions are always satisfied. Let Q_q, $Q_{q^{\cdot}}$, and Q_ξ be bounded sets of admissible values of phase variables q, and $q^{\cdot}$, and of parameters ξ.

We shall consider two kinds of relationship z between the m-vector z of the considered variables related to the functioning of system (1.1), and q

$$z = f(q) \tag{1.2}$$

$$z^{\cdot} = f(z, q, q^{\cdot}) \tag{1.3}$$

The relationships (1.2) and (1.3) are usually referred to as holonomic and nonholonomic, respectively.

Strictly speaking, the correct form of equations of motion of systems with nonholonomic relationship (1.3) require the introduction of variables z and $z^{\cdot}$ to the arguments of the matrix function $A\ (\cdot)$ and vector function $b\ (\cdot)$. However for the sake of shortness of formulas this is omitted, since all subsequent reasoning is essentially independent of that requirement.

An example of a mechanical system with a holonomic relation is the multilink manipulator [2, 3]. In it vector q defines the manipulator configuration (components of are angles between links, etc.), and z defines the position of its gripping mechanism (components of z are the coordinates and, possibly, directional cosines of the gripper). A trolley on wheels or on a tracked undercarriage provides an example of a mechanical system with nonholonomic relationship. In that case the components of q are the angles of turn of the wheels or of the sprockets driving the tracks, and the components of z are Cartesian coordinates of the trolley and its heading angle.

The programed motion will be defined by the vector function $z_p \equiv z_p(t), t \geqslant t_0$, such that function $q_p\ (t)$, which is the solution of Eqs. (1.2) or (1.3), after the substitution $z = z_p(t)$ and $z^{\cdot} = z_p^{\cdot}\ (t)$ satisfies conditions

1) $q_p(t) \in Q_q^{\delta_1},\quad q_p^{\cdot}(t) \in Q_{q^{\cdot}}^{\delta_2}\quad \forall t \geqslant t_0$;

2) $q_p^{\cdot\cdot}\ (t)$ is piecewise continuous and $\|\ q_p^{\cdot\cdot}\ (t)\ \| \leqslant c_{q^{\cdot\cdot}}\ \forall t \geqslant t_0$, where $Q_q^{\delta_1} \subset Q_q$, $Q_{q^{\cdot}}^{\delta_2} \subset Q_{q^{\cdot}}$, and the distance between the boundaries of sets $Q_q^{\delta_1}$ and $Q_{q^{\cdot}}^{\delta_2}$ and the boundaries of sets Q_q and $Q_{q^{\cdot}}$ is, respectively equal δ_1 and δ_2 . The positive parameters δ_1 and δ_2 are later selected on grounds of the necessity to ensure phase constraints on the real motion for all $t \geqslant t_0$.

Note. 1°. Usually $n > m$, i.e. Eqs. (1.2) or (1.3) are nonuniquely solvable for q . This ambiguity (the system kinematic redundancy) can be used for satisfying certain supplementary conditions (bypassing obstacles, selection of optimal $q_p\ (t)$ for specified $z_p\ (t)$, etc.). When $n < m$, a part of components of vector z is a function of the remaining ones and can be discarded.

2°. Phase constraints can be reformulated in terms of z (by using the equations of relation (1.2) or (1.3)). Namely, $z \in Q_z$ and $z^{\cdot} \in Q_{z^{\cdot}}$. The programed motion $z_p\ (t)$ must satisfy conditions $z_p\ (t) \in Q_z^{\delta_3}$, $z_p^{\cdot}\ (t) \in Q_{z^{\cdot}}^{\delta_4}$, $z_p^{\cdot\cdot}\ (t)$ must be piecewise continuous, and $\|\ z_p^{\cdot\cdot}\ (t)\ \| \leqslant c_{z^{\cdot\cdot}}$ for all $t \geqslant t_0$.

3°. The definition of programed motion must include the stipulation that the phase constraints on programed motions must be satisfied with some " margins " δ_1 and δ_2. Below we adduce formulas for calculating δ_1 and δ_2 using a priori known parameters and the discrepancy between real and programed motions at the initial instant of time. In practice the selection of programed motion may be, for instance, as follows: select a function to define the programed motion, measure the initial discrepancy (the actual initial state of the system is always assumed known), calculate margins δ_1 and δ_2 with which phase constraints must be satisfied, and check that this is so in the case of the selected function for all $t \geqslant t_0$. If the constraints are satisfied that function is taken as the programed motion. Otherwise, another function with smaller initial discrepancy is selected, and the described procedure repeated.

In the scheme considered here the programed motion is assumed to be specified and to satisfy all necessary conditions.

The aim of the control is to realize the programed motion of the system with the best possible exactness. If $z_p(t_0) = z(t_0)$, $z_p^{\cdot}(t_0) = z^{\cdot}(t_0)$ and $q_p(t_0) = q(t_0)$, and the initial parameters of the equation of motion are known, the programed control law $u_p(t) = A(q_p, q_p^{\cdot}, \xi)\, q_p^{\cdot\cdot} + b(q_p, q_p^{\cdot}, \xi)$ ensure the required motion, i.e. the actual motion $z(t)$ conforms to the programed one $z_p(t)$ for all $t \geqslant t_0$. However under real conditions parameters ξ of the equation of motion can only be known with an accuracy to within their belonging to the specified set Q_ξ. Furthermore initial perturbations are present, i.e. $z(t_0) \neq z_p(t_0)$ and $z^{\cdot}(t_0) \neq z_p^{\cdot}(t_0)$. Hence it is not possible to use the programed control $u_p(t)$.

The problem of adaptive stabilization of programed motion consists of synthesizing the control law $u = U(q, q^{\cdot}, z, z^{\cdot}, t)$ in the class of piecewise continuous bounded functions which for any parameters $\xi \in Q_\xi$ and any initial perturbations ensure the transition of the system to the programed motion within specified accuracy beginning at a certain instant of time $t_* \geqslant t_0$

$$\| z(t) - z_p(t) \| < \varepsilon_1, \ \| z^{\cdot}(t) - z_p^{\cdot}(t) \| \leqslant \varepsilon_2, \ \forall t \geqslant t_* \qquad (1.4)$$
$$\forall \xi \in Q_\xi, \ \varepsilon_1, \ \varepsilon_2 > 0$$

The time $t_* - t_0, t_* = t_*(\varepsilon_1, \varepsilon_2, \xi)$, is called the time of the adaptation transition process in the sense of the specific condition (1.4).

The formulated problem is solved in two stages. First, on the assumption that parameters ξ are known, the stabilizing laws of control, different from those proposed earlier [6-8], are derived. Then, on the basis of such nonadaptive control scheme, the adaptive problem is solved by reducing it to the solution of a system of inequalities for the parameters of the sought control by the method of finite-convergent algorithms proposed in [1].

2. Stabilization of programed motions in the nonadaptive case. Let us assume that parameters ξ of the equation of motion (1.1) are known. We shall, first, consider mechanical systems with holonomic relationship defined by (1.1) and (1.2), and devise the stabilizing control on the basis of the stipulation that the difference between real and programed motions $e_z(t) = z(t) - z_p(t)$ or $e_q(t) = q(t) - q_p(t)$ must satisfy the differential equation of discrepancies

$$e^{\cdot\cdot} = \Gamma_1 e^{\cdot} + \Gamma_2 e \qquad (2.1)$$

where Γ_1 and Γ_2 are constant matrices such that the zero solution of this equation is asymptotically stable.

The discrepancy equation can generally be selected from the class of second order nonlinear differential equations which have a trivial solution and ensure the specified quality of the transition process. In the described variant the selection of matrices Γ_1 and Γ_2 (and later of B_1 and B_2) depends on the specification of the transition process, and is carried out by methods of the theory of linear differential equations, in particular these matrices may be selected on optimization considerations.

The normal form of Eq. (2.1) is

$$E^{\cdot} = \Gamma E, \quad E = \begin{Vmatrix} e \\ e^{\cdot} \end{Vmatrix}, \quad \Gamma = \begin{Vmatrix} 0 & I \\ \Gamma_1 & \Gamma_2 \end{Vmatrix}$$

where 0 is a zero matrix and I is a unit matrix of corresponding dimension. We assume that the eigenvalues γ_i of matrix Γ, which is stable, have a unit multiplicity. Then the estimate

$$\| E(t) \| \leqslant C \| E(t_0) \| \exp[-\gamma(t - t_0)], \quad \gamma = -\max_i \{\operatorname{Re} \gamma_i\} \tag{2.2}$$

where C is a positive constant dependent on the selection of matrices Γ_1 and Γ_2, is valid.

In the considered holonomic case we stipulate that the quantity $e_q(t)$ must satisfy the discrepancy equation (2.1). We select the margins

$$\delta_1 = \delta_2 = C \| E_q(t_0) \|, \quad E_q(t) = (e_q(t), e_q^{\cdot}(t))^T$$

Theorem 1. If function $f(q)$ in (1.2) and its first derivative with respect to time $f_1(q, q^{\cdot}) \equiv (df/dq)\, q^{\cdot}$ belongs to Lipschitz's classes with respect to its arguments with constants L, L_q, and $L_{q^{\cdot}}$, respectively, then the control law

$$u = A(q, q^{\cdot}, \xi)[q_p^{\cdot\cdot} + \Gamma_1(q^{\cdot} - q_p^{\cdot}) + \Gamma_2(q - q_p)] + b(q, q^{\cdot}, \xi) \tag{2.3}$$

ensures the fulfilment of condition (1.4), and for the transition process time the estimate

$$t_* - t_0 < \frac{1}{\gamma} \max\left\{\ln \frac{CL\|E_q(t_0)\|}{\varepsilon_1}, \quad \ln \frac{C(L_q + L_{q^{\cdot}})\|E_q(t_0)\|}{\varepsilon_2}\right\} \tag{2.4}$$

is valid.

Proof. The control (2.3) is obviously piecewise continuous and bounded. Let us close by this control the system (1.1). Owing to the reversibility of matrix $A(\cdot)$ we have

$$e_q^{\cdot\cdot} = \Gamma_1 e_q^{\cdot} + \Gamma_2 e_q$$

Since $\| e_q \| \leqslant \| E_q \|$ and $\| e_q^{\cdot} \| \leqslant \| E_q \|$, hence taking into account (2.2) we obtain

$$\left.\begin{array}{l} \| q(t) - q_p(t) \| \\ \| q^{\cdot}(t) - q_p^{\cdot}(t) \| \end{array}\right\} \leqslant C \| E_q(t_0) \| \exp[-\gamma(t - t_0)]$$

The actual motion $q(t)$ does not fall outside the admissible sets Q_q and $Q_{q^{\cdot}}$, since it does not diverge from the programed motion by more than $C \| E_q(t_0) \|$.

The estimates

$$\| z(t) - z_p(t) \| \leqslant LC \| E_q(t_0) \| \exp[-\gamma(t - t_0)]$$
$$\| z^{\cdot}(t) - z_p^{\cdot}(t) \| \leqslant C(L_q + L_{q^{\cdot}}) \| E_q(t_0) \| \exp[-\gamma(t - t_0)]$$

are evidently valid.

Solution of the system of inequalities

$$LC \| E_q (t_0) \| \exp [-\gamma' (t - t_0)] < \varepsilon_1$$
$$C (L_q + L_{q^\cdot}) \| E_q (t_0) \| \exp [-\gamma (t - t_0)] < \varepsilon_2, \quad \varepsilon_1, \varepsilon_2 > 0$$

yields

$$t - t_0 > \frac{1}{\gamma} \max \left\{ \ln \frac{CL \| E_q (t_0) \|}{\varepsilon_1}, \quad \ln \frac{C (L_q + L_{q^\cdot}) \| E_q (t_0) \|}{\varepsilon_2} \right\}$$

from which directly follows the required estimate (2.4).

Let us consider the nonholonomic system whose generalized coordinates z and q are related by Eqs. (1.1) and (1.3). In this case the closeness of $z (t)$ and $z_p (t)$ to $z^\cdot (t)$ and $z_p^\cdot (t)$ in the sense of (1.4) does not generally follow from the closeness of $q (t)$ and $q_p (t)$ to $q^\cdot (t)$ and $q_p^\cdot (t)$. Hence the control law (2.3) determined by Theorem 1 does not ensure the attainment of the specified conditions (1.4) for systems with the nonholonomic relationship (1.3).

Unlike in the holonomic case in constructing the control, we aim here at subjecting the quantity $e_z (t)$ to the discrepancy equation (2.1). We introduce the notation

$$H (t, \Gamma_1, \Gamma_2) = z_p^{\cdot\cdot} + \Gamma_1 (z^\cdot - z_p^\cdot) + \Gamma_2 (z - z_p), \quad E_z = \left\| \begin{matrix} e_z \\ e_z^\cdot \end{matrix} \right\| \tag{2.5}$$

and select the margins $\delta_3 = \delta_4 = C \| E_z (t_0) \|$.

L e m m a 1. The vector function $q_\gamma (t)$ selected for $t \geqslant t_0$ on the basis of condition

$$\int_{t_0}^{t} H (t, \Gamma_1, \Gamma_2) dt + z^\cdot (t_0) = f (z, q_\gamma, q_\gamma^\cdot) \tag{2.6}$$
$$q_\gamma (t_0) = q (t_0), \quad q_\gamma^\cdot (t_0) = q^\cdot (t_0)$$

satisfies phase constraints and ensures the fulfilment of conditions

$$\lim \| z (t) - z_p (t) \| = 0, \quad \lim \| z^\cdot (t) - z_p^\cdot (t) \| = 0, \quad t \to \infty \tag{2.7}$$

and consequently, also, of conditions (1.4).

N o t e 4°. For actual mechanical systems Eqs. (1.3) or (2.6) have unique solutions for variables q (or part of these when $n > m$). If these equations are algebraically solvable for $q^\cdot$ (which is not always so), a numerical solution can be obtained by one of the conventional methods. However even in the case of unsolvability (as in the example considered below) it is usually possible, with allowance for the particular properties of Eqs. (1.3) or (2.6), to obtain a solution; function $q_\gamma^{\cdot\cdot} (t)$ is assumed to be piecewise continuous, which usually happens when $z_p^{\cdot\cdot} (t)$ is piecewise continuous.

P r o o f. From (1.3) and (2.6) we have

$$z^\cdot = \int_{t_0}^{t} H (t, \Gamma_1, \Gamma_2) dt + z^\cdot (t_0)$$

and after differentiation with respect to time

$$z^{\cdot\cdot} = H(t, \Gamma_1, \Gamma_2) = z_p^{\cdot\cdot} + \Gamma_1 (z^{\cdot} - z_p^{\cdot}) + \Gamma_2 (z - z_p)$$

or $e_z^{\cdot\cdot} = \Gamma_1 e_z^{\cdot} + \Gamma_2 e_z$. By virtue of (2.2) and (2.6) we obtain

$$\left.\begin{array}{l} \| z(t) - z_p(t) \| \\ \| z^{\cdot}(t) - z_p^{\cdot}(t) \| \end{array}\right\} \leqslant C \| E_z(t_0) \| \exp[-\gamma(t - t_0)] \tag{2.8}$$

from which follow conditions (2.7) and consequently, also, (1.4). Actual phase variables z and $z^{\cdot}$ do not deviate from the programed by more than $C \| E_z(t_0) \|$, hence the phase constraints are not violated. In terms of q the motion $q_\gamma(t)$, which in conformity with the previous statement does not fall outside the admissible sets, corresponds to that motion.

Theorem 2. The control law

$$u = A(q, q^{\cdot}, \xi) q_\gamma^{\cdot\cdot} + b(q, q^{\cdot}, \xi) \tag{2.9}$$

where $q_\gamma(t)$ is selected in accordance with Lemma 1, ensures that all conditions $q(t) = q_\gamma(t)$ are satisfied for all $t \geqslant t_0$, and consequently guarantees the fulfilment of conditions (1.4).

Proof. We substitute control (2.9) for system (1.1). Owing to the reversibility of matrix $A(\cdot)$ for all $t \geqslant t_0$ we obtain $q_\gamma^{\cdot\cdot}(t) = q^{\cdot\cdot}(t)$. Integrating twice and taking into account the coincidence of initial conditions we satisfy the requirement. Piecewise continuity and boundedness of control (2.9) is checked by obvious means.

Let us estimate the time of the transition process. By solving the equality

$$C \| E_z(t_0) \| \exp[-\gamma(t - t_0)] < \varepsilon, \quad \varepsilon > 0$$

we obtain

$$t > t_*, \quad t_* = t_0 + \frac{1}{\gamma} \ln \frac{C \| E_z(t_0) \|}{\varepsilon}$$

Taking into account (2.8) we find that conditions (1.4) are satisfied at least from the instant of time t_* when $\varepsilon_1 = \varepsilon_2 = \varepsilon$.

3. Adaptive stabilization of programed motions of mechanical systems. Let us consider the case of practical importance in which parameters ξ in the equation of motion (1.1) are unknown. In that case it is not possible to use control laws (2.3) and (2.9), and it is necessary to construct an adaptive control.

We assume that the following conditions are satisfied:

1) the equation of motion is of the form

$$G(q, q^{\cdot}, q^{\cdot\cdot}) \tau(\xi) = u \tag{3.1}$$

where $G(\cdot)$ is an $n \times N$ matrix function and $\tau(\xi)$ is an N-vector, and

2) the set $Q_\tau \equiv \tau(Q_\xi)$ is convex in R^N.

Let us consider the nonholonomic system (1.1), (1.3) (the holonomic system (1.1), (1.2) is analyzed in the same way).

We define the control law by the formula

$$u = G[q, q^{\cdot}, q_\gamma^{\cdot\cdot} + B_1(q^{\cdot} - q_\gamma^{\cdot}) + B_2(q - q_\gamma)]\tau_k, \quad t \in (t_k, t_{k+1}] \tag{3.2}$$

where B_1 and B_2 are certain $n \times n$ matrices, τ_k is an N- vector to be taken as the estimate of the unknown vector $\tau(\xi)$ in the time interval $(t_k, t_{k+1}]$, $k = 0$, $1 \ldots$; and t_k are the instants of vector τ corrections, defined below. The control (3.2) is obviously piecewise continuous and bounded.

Let us consider the subsidiary inequalities

$$\Phi(\tau, \tau_k, t) \equiv \varepsilon_u - \| u - G(q, q^{\cdot}, q^{\cdot\cdot})\tau \| > 0, \quad \varepsilon_u > 0 \tag{3.3}$$

which are obviously solvable with margin ε_u for $\tau = \tau(\xi)$. Hence the finite-convergent algorithms of the form

$$\tau_{k+1} = \begin{cases} \tau_k, \text{ если } \Phi(\tau_k, \tau_k, t) > 0 \\ T[\tau_k, \Phi(\tau_k, \tau_k, t_k')] \end{cases} \tag{3.4}$$

where t_k' is the first instant of time in the interval $(t_k, t_{k+1}]$ such that $\Phi(\tau_k, \tau_k, t_k') \leqslant 0$ and τ_0 is an arbitrary N- vector Q_τ of the initial approximation, can be used for solving the above inequalities (for further details see [1-4]). These algorithms, called adaption algorithms, guarantee that (3.3) is satisfied for all $t \geqslant t_r$ and $\tau = \tau_r = \text{const}$, after the finite number r of violation of inequalities (3.3) and, consequently also after r corrections of τ in accordance with (3.4). In other words, the estimate τ_k of vector $\tau(\xi)$ becomes "frozen" from a certain finite instant of time, and inequalities (3.3) are thereon satisfied.

We denote by θ the time required for calculating τ_{k+1} on the basis of τ_k by (3.4). Then obviously $t_{k+1} = t_k' + \theta$, and inequality (3.3) is valid for $t \in U_k(t_k, t_{k+1} - \theta]$ and violated for $t \in U_k(t_{k+1} - \theta, t_{k+1}$.

An example of the recurrent finite-converging algorithm is (3.4) with

$$T[\tau_k, \varphi(\tau_k, \tau_k, t_k')] = P_{Q_\tau}[\tau_k + G_k^T(G_kG_k^T)^{-1}(u(t_k') - G_k\tau_k)] \tag{3.5}$$
$$\tau_0 \in Q_\tau, \quad G_k = G[q(t_k'), q_k^{\cdot}(t_k'), q^{\cdot\cdot}(t_k')]$$

where P_{Q_τ} is the operator of orthogonal projection onto set Q_τ. It can be shown that for r corrections of algorithm (4.3), (3.5) the estimate

$$r \leqslant \frac{\| \tau(\xi) - \tau_0 \|^2 C_G^2}{\varepsilon_u^2}, \quad C_G = \sup \| G(\cdot) \| \tag{3.6}$$

is valid.

All quantities in the left-hand side of inequalities are bounded, hence there exists a positive number Δ such that

$$\| u - G(q, q^{\cdot}, q^{\cdot\cdot})\tau \| < \Delta, \quad t \geqslant t_0$$

Below we shall need the following lemma.

Lemma 2. Let matrix

$$\Gamma = \begin{Vmatrix} 0 & I \\ \Gamma_1 & \Gamma_2 \end{Vmatrix}$$

be stable and its eigenvalues γ_i have a unitary multiplicity. We assume that the vector function $\eta(t)$ is piecewise continuous in $[t_0, \infty)$ and satisfies the condition

$$\| \eta(t) \| \leqslant \begin{cases} \delta, & t \in [t_0, \infty) \setminus F \\ \Delta, & t \in F \end{cases} \tag{3.7}$$

and that the Lebesgue measure $\mu(F) < \nu$, where δ, Δ, ν are some positive numbers. Then for the solution of equation

$$e^{\cdot\cdot} = \Gamma_1 e^{\cdot} + \Gamma_2 e + \eta(t)$$

with initial data $e(t_0)$ and $e^{\cdot}(t_0)$ the following estimate is valid:

$$\left.\begin{matrix} \| e(t) \| \\ \| e^{\cdot}(t) \| \end{matrix}\right\} \leqslant C \exp[-\gamma(t - t_0)] \| s_0 \| + \frac{C}{\gamma}\delta + C\Delta\nu$$

$$s_0 = (e(t_0), e^{\cdot}(t_0))^T, \quad \gamma = -\max_i \{\operatorname{Re} \gamma_i\}$$

where C is some positive constant.

The proof of Lemma 2 is trivial (see, e.g., [7]).

Condition 1) and formulas (3.2) imply that

$$u = A(q, q^{\cdot}, \xi_k)[q_\gamma^{\cdot\cdot} + B_1(q^{\cdot} - q_\gamma^{\cdot}) + B_2(q - q_\gamma)] + b(q, q^{\cdot}, \xi_k)$$

where vector $\xi_k \in Q_\xi$ is such that $\tau(\xi_k) = \tau_k \in Q$.

We have

$$v - u = A(q, q^{\cdot}, \xi_k)[q^{\cdot\cdot} - q_\gamma^{\cdot\cdot} - B_1(q^{\cdot} - q_\gamma^{\cdot}) - B_2(q - q_\gamma)]$$
$$v = G(q, q^{\cdot}, q^{\cdot\cdot})\tau_k \equiv A(q, q^{\cdot}, \xi_k) q^{\cdot\cdot} + b(q, q^{\cdot}, \xi_k)$$

If $e_q(t) = q(t) - q_\gamma(t)$, then

$$e_q^{\cdot\cdot} = B_1 e_q^{\cdot} + B_2 e_q + A^{-1}(q, q^{\cdot}, \xi_k)(v - u), \quad e_q(t_0) = e_q^{\cdot}(t_0) = 0 \tag{3.8}$$

and by virtue of (3.2)-(3.5) the estimate

$$\| A^{-1}(q, q^{\cdot}, \xi_k)(v - u) \| \leqslant \begin{cases} C_A \varepsilon_u, & t \in [t_0, \infty) \setminus F \\ C_A \Delta, & t \in F = \bigcup_{k=1}^{l} (t_k - \theta, t_k] \end{cases} \tag{3.9}$$

$$C_A = \sup \| A^{-1}(\cdot) \|, \quad l \leqslant r$$

is valid.

It is obvious that the Lebesgue measure $\mu(F) \leqslant r\theta$.

We assume that matrix

$$B = \begin{Vmatrix} 0 & I \\ B_1 & B_2 \end{Vmatrix}$$

is stable and its eigenvalues have unitary multiplicity, then, applying Lemma 2 to Eq. (3.8) and taking into account estimate (3.8), we obtain

$$\left.\begin{matrix} \| e_q(t) \| \\ \| e_q^{\cdot}(t) \| \end{matrix}\right\} \leqslant \frac{C_q}{\gamma_q} C_A \varepsilon_u + C_A C_q \Delta r\theta \tag{3.10}$$

From (3.8) we have

$$\| e_q^{\cdot\cdot}(t) \| \leqslant C_q C_A \left(\frac{\varepsilon_u}{\gamma_q} + \Delta r\theta \right) (\| B_1 \| + \| B_2 \|) + \| \eta(t) \| \tag{3.11}$$

$$\eta(t) = A^{-1}(q, q^{\cdot}, \xi_k)(v - u)$$

Differentiating conditions (2.6) and (1.3) with respect to time we obtain

$$H(t, \Gamma_1, \Gamma_2) = \frac{d}{dt} f(z, q_\gamma, q_\gamma^{\cdot}) = \frac{\partial f}{\partial z} z^{\cdot} + \frac{\partial f}{\partial q} q_\gamma^{\cdot} + \frac{\partial f}{\partial q^{\cdot}} q_\gamma^{\cdot\cdot} \equiv f_1(z, z^{\cdot}, q_\gamma, q_\gamma^{\cdot}, q_\gamma^{\cdot\cdot})$$

$$z^{\cdot\cdot} = f_1(z, z^{\cdot}, q, q^{\cdot}, q^{\cdot\cdot})$$

Let us assume that function $f_1(\cdot)$ belongs to Lipschitz's class with respect to q, $q^{\cdot}$, and $q^{\cdot\cdot}$ with constants L_q, $L_{q^{\cdot}}$, and $L_{q^{\cdot\cdot}}$, respectively. Then, taking into account (2.5), (3.10), and (3.11), we obtain

$$\| z^{\cdot\cdot} - z_p^{\cdot\cdot} - \Gamma_1(z^{\cdot} - z_p^{\cdot}) - \Gamma_2(z - z_p) \| = \| f_1(z, z^{\cdot}, q, q^{\cdot}, q^{\cdot\cdot}) - f_1(z, z^{\cdot}, q_\gamma, q_\gamma^{\cdot}, q_\gamma^{\cdot\cdot}) \| \leqslant L_q \| q(t) - q_\gamma(t) \| + L_{q^{\cdot}} \| q^{\cdot}(t) - q_\gamma^{\cdot}(t) \| + L_{q^{\cdot\cdot}} \| q^{\cdot\cdot}(t) - q_\gamma^{\cdot\cdot}(t) \| \leqslant (L_q + L_{q^{\cdot}} + L_{q^{\cdot\cdot}} \times (\| B_1 \| + \| B_2 \|)) \left(\frac{C_q}{\gamma_q} C_A \varepsilon_u + C_q C_A \Delta r\theta \right) + L_{q^{\cdot\cdot}} \| \eta(t) \|$$

We introduce the notation

$$e_z(t) = z(t) - z_p(t)$$

$$(L_q + L_{q^{\cdot}} + L_{q^{\cdot\cdot}}(\| B_1 \| + \| B_2 \|)) \left(\frac{C_q}{\gamma_q} C_A \varepsilon_u + C_q C_A \Delta r\theta \right) = K(\varepsilon_u, r, \theta)$$

and obtain

$$e_z^{\cdot\cdot} = \Gamma_1 e_z^{\cdot} + \Gamma_2 e_z + \zeta(t)$$

$$\| \zeta(t) \| \leqslant \begin{cases} K(\varepsilon_u, r, \theta) + L_{q^{\cdot\cdot}} C_A \varepsilon_u, & t \in [t_0, \infty) \setminus F \\ K(\varepsilon_u, r, \theta) + L_{q^{\cdot\cdot}} C_A \Delta, & t \in F \equiv \bigcup_{k=1}^{l} (t_k - \theta, t_k] \end{cases}$$

Applying once again Lemma 2 we obtain the estimate

$$\left.\begin{array}{l}\| z(t) - z_p(t) \| \\ \| z^{\cdot}(t) - z_p^{\cdot}(t) \|\end{array}\right\} \leqslant C_z \| E_z(t_0) \| \exp[-\gamma_z(t - t_0)] + \Psi(\varepsilon_u, r, \theta) \tag{3.12}$$

$$\Psi(\varepsilon_u, r, \theta) = \frac{C_z}{\gamma_z}(L_{q^{\cdot\cdot}} C_A \varepsilon_u + K(\varepsilon_u, r, \theta)) + C_z r\theta(L_{q^{\cdot\cdot}} C_A \Delta + K(\varepsilon_u, r, \theta))$$

$$E_z(t_0) = (e_z(t_0), e_z^{\cdot}(t_0))^T$$

This shows that the considered adaptive control can ensure the fulfilment of (1.4) when $\varepsilon_1 = \varepsilon_2 = \varepsilon > \Psi$. If the actual motion $z(t)$ is to satisfy phase constraints, it is necessary to stipulate that these constraints must be satisfied by the programed motion $z_p(t)$ with margins $\delta_3 = \delta_4 = C_z \| E_z(t_0) \| + \Psi$.

Let us estimate the time $t_* - t_0$ of the adaptation transition process. Solving the inequality

$$C_z \| E_z(t_0) \| \exp[-\gamma_z(t - t_0)] + \Psi < \varepsilon, \quad \varepsilon > \Psi$$

with allowance for (3.12), we obtain

$$t_* - t_0 < \frac{1}{\gamma_z} \ln \frac{C_z \| E_z(t_0) \|}{\varepsilon - \Psi} \tag{3.13}$$

In this way the following theorem has been proved.

Theorem 3. Assume that conditions 1) and 2) are satisfied and that matrix B with simple eigenvalues is stable, and that function $f_1(\cdot)$ belongs to Lipschitz's class with respect to q, $q^{\cdot}$ and $q^{\cdot\cdot}$ with constants L_q, $L_{q^{\cdot}}$ and $L_{q^{\cdot\cdot}}$, respectively. Let also the phase constrictions on $z_p(t)$ be satisfied with margins $\delta_3 = \delta_4 = C_z \| E_z(t_0) \| + \Psi$. Then the control (3.2)-(3.5) ensures the fulfilment of conditions (1.4) with $\varepsilon_1 = \varepsilon_2 = \varepsilon > \Psi$, and the estimate (3.15) of the adaption transition process is valid.

In practice the time of the adaption transition process must be as short as possible. This can be achieved by the optimal selection of parameters ε_u and θ (which defines the time-optimal action of algorithm (3.4), (3.5)) with the aim of minimizing the right-hand side of inequality (3.12). We substitute estimate (3.6) for the number r of corrections. Then, taking into account that $\partial\Psi / \partial\theta > 0$, we take the shortest possible time $\theta = \theta_*$ (limited by technical characteristics of the adaption algorithm). After this we select $\varepsilon_u^* > 0$ on the basis of minimization of function $\Psi(\varepsilon_u, \theta_*)$. Such ε_u^* exists and is unique, since $\Psi(\varepsilon_u, \theta_*)$ as a function of ε_u is of the form

$$\Psi(\varepsilon_u, \theta_*) = \alpha_1 \varepsilon_u + \frac{\alpha_2}{\varepsilon_u} + \frac{\alpha_3}{\varepsilon_u^2} + \frac{\alpha_4}{\varepsilon_u^4}, \quad \alpha_i > 0$$

Thus the optimal estimate of the time of the adaptation transition process for $\varepsilon > \Psi(\varepsilon_u^*, \theta_*)$ is of the form

$$t_* - t_0 < \frac{1}{\gamma_z} \ln \frac{C_z \| E_z(t_0) \|}{\varepsilon - \Psi(\varepsilon_u^*, \theta_*)}$$

In the particular case when $\| E_z(t_0) \| = 0$ and $\varepsilon > \Psi(\varepsilon_u^*, \theta_*)$ the time of adaptation transition process is zero, i. e. conditions (1.4) are satisfied from the initial instant of time $t = t_* = t_0$ due to the adaptive tuning of parameters τ_k of the control law (3.2) in conformity with (3.4) and (3.5).

4. Examples. Let us consider a transportation robot in the shape of a self-propelled trolley on tracks, for which the equations of nonholonomic relationships (1.3) are (with some degree of idealization) of the form

$$x^{\cdot} = \frac{r}{2}(\varphi_1^{\cdot} + \varphi_2^{\cdot}) \cos \psi, \quad y^{\cdot} = \frac{r}{2}(\varphi_1^{\cdot} + \varphi_2^{\cdot}) \sin \psi \tag{4.1}$$

$$\psi^{\cdot} = \frac{r}{2l}(\varphi_1^{\cdot} - \varphi_2^{\cdot}), \quad \varphi_1(t_0) = \varphi_2(t_0) = 0, \quad \psi(t_0) = \psi_0$$

where x, y are Cartesian coordinates of the middle of the axis with the driving sprockets of tracks, ψ is the angle of the trolley course, φ_1, φ_2 are the angles of rotation of driving sprockets of radius r, and l is the trolley half-base. Integrating the third of Eqs. (4.1) and substituting the result into the first and second of these, we obtain

$$\begin{Bmatrix} x^{\cdot} \\ y^{\cdot} \end{Bmatrix} = \frac{r}{2}(\varphi_1^{\cdot} + \varphi_2^{\cdot}) \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} \left[\frac{r}{2l}(\varphi_1 - \varphi_2) + \psi_0\right] \tag{4.2}$$

Hence $z = (x, y)^T$ and $q = (\varphi_1, \varphi_2)^T$. Equations (4.2) cannot be solved algebraically for $\varphi_1^{\cdot}$ and $\varphi_2^{\cdot}$ it is, however, possible to express these quantities in terms of variables z (see Note 4°), namely

$$\varphi_{1,2}^{\cdot} = \frac{1}{r}\left(\sqrt{x^{\cdot 2} + y^{\cdot 2}} \pm l \frac{y^{\cdot\cdot} x^{\cdot} - y^{\cdot} x^{\cdot\cdot}}{x^{\cdot 2} + y^{\cdot 2}}\right)$$

Hence formulas (4.2) satisfy the conditions of Lemma 1. The form of these shows that they belong to nonholonomic Chaplygin systems. The equations of motion are of the form (1.1), where $A(\cdot) = A(\xi)$ is constant nonsingular 2×2 matrix whose elements depend on the mass, moments of inertia, and linear dimensions of various parts of the trolley, $b(\cdot) = b(q^{\cdot}, \xi)$ is a 2-vector function which depends on $q^{\cdot\cdot}$ and parameters ξ which in addition to the previously indicated contain coefficients of (internal and external) friction, and $u = (u_1, u_2)^T$ is the 2-vector of control moments. The equations of motion and relationship satisfy stipulations of Theorem 3, consequently, the derived above algorithms may be used for calculating the adaptive stabilization of programed motions a transport trolley.

Similar algorithms may, also, be used for the adaptive stabilization of programed motions of a robot manipulator which is a system with holonomic relationships. A detailed exposition of solution of that problem are given in [2-4] together with experimental results of its simulation on a computer.

REFERENCES

1. Iakubovich, V. A., Finite convergent algorithms for solving denumerable systems of inequalities and their application in problems of synthesis of adaptive systems. Dokl. akad. Nauk SSSR, Vol. 189, №3, 1969.
2. Gusev, S. V., Timofeev, A. V. and Iakubovich, V. A., On one hierarchical system of integral robot control. 4-th Internat. Joint Conference on Artificial Intellect, Vol. 9. Algorithms for the control of motion and work. Moscow, VINITI, 1975.
3. Timofeev, A. V. and Iakubovich, V. A., Adaptive control of programed motion of a robot manipulator. Collection: Problems of Cybernetics. Additive Systems. Moscow, "Nauka", 1976.
4. Timofeev, A. V. and Ekalo, Iu. V., Stability and stabilization of programmed motions of a robot manipulator. Avtomatika i Telemekhanika, №10, 1976.
5. Aksenov, G. S. and Fomin, V. N., On the problem of adaptive control of a manipulator. Collection: Problems of Cybernetics. Adaptive Systems. Moscow, "Nauka", 1976.
6. Zubov, V. I., Lectures on the Theory of Control. Moscow, "Nauka", 1975.
7. Barbashin, E. A., Introduction to the Theory of Stability. Moscow, "Nauka", 1967.
8. Krasovskii, N. N., The Theory of Motion Control. Moscow, "Nauka", 1968.

Translated by J.J. D.